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INFLUENCE OF FINITE ION LARMOR RADIUS AND EQUILIBRIUM SELF ELEC--ETC(U)
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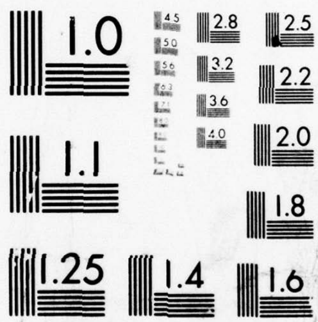

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INFLUENCE OF FINITE ION LARMOR RADIUS AND EQUILIBRIUM SELF ELECTRIC FIELDS ON THE ION RESONANCE INSTABILITY

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The influence of finite ion Larmor radius and equilibrium self-electric fields on the ion resonance instability in a nonneutral plasma column is examined, and a closed algebraic dispersion relation for the complex eigenfrequency ω is obtained. It is shown that finite ion Larmor radius effects can have a strong stabilizing influence for azimuthal mode numbers $\ell \geq 2$, particularly when the equilibrium self-electric field is sufficiently weak.

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†On leave of absence from the University of Maryland, College Park, Md.

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I. INTRODUCTION

One of the most basic instabilities that characterizes a nonneutral plasma[1] with both ion and electron components is the ion resonance instability.[2-5] In cylindrical geometry, the ion resonance instability can be described as a two-rotating-stream instability[2] in which the relative rotation between electrons and ions is produced by the equilibrium self-electric field $E_r^0(r)\hat{e}_r$. Previous analyses of this instability have been based on macroscopic cold-fluid models[2-4] in which the ion and electron motion is assumed to be laminar. In this paper, we investigate the influence of finite ion Larmor radius and equilibrium self-electric field effects on the ion resonance instability in a nonneutral plasma column with rectangular electron and ion density profiles. The analysis is carried out within the framework of a hybrid Vlasov-fluid model. The electrons are described as a macroscopic, cold ($T_e \rightarrow 0$) fluid immersed in a uniform axial magnetic field $B_0\hat{e}_z$. On the other hand, to allow for the possibility of large ion orbits with thermal Larmor radius comparable to the radius of the plasma column, we adopt a fully kinetic model for the ions in which the ions are described by the Vlasov equation. An important conclusion of the present analysis is that the growth rate of the ion resonance instability exhibits a very sensitive dependence on $\hat{r}_{Li}/R_p$, $\hat{\omega}_{pe}^2/\omega_{ce}^2$ and f. (Here $\hat{r}_{Li}$ is the characteristic thermal ion Larmor radius, R_p is the radius of the plasma column, $\hat{\omega}_{pe}$ is the electron plasma frequency, ω_{ce} is the electron cyclotron frequency, and $f=n_i^0/n_e^0$ is the fractional charge neutralization provided by the positive ions.)

For azimuthally symmetric electron equilibria ($\partial/\partial\theta=0$ and $\partial/\partial z=0$) characterized by electron density $n_e^0(r)$ and mean velocity $\mathbf{V}_e^0(\mathbf{x})=V_{e\theta}^0(r)\hat{e}_\theta$, equilibrium force balance on an electron fluid element in the radial direction can be expressed as $-m_e V_{e\theta}^{02}(r)/r=-eE_r^0(r)-eV_{e\theta}^0(r)B_0/c$, or equivalently $\omega_e^2-\omega_{ce}\omega_e+\omega_E\omega_{ce}=0$, where $\omega_{ce}=eB_0/m_ec$, $\omega_e(r)=V^0_{e\theta}(r)/r$, and $\omega_E(r)$ is the angular $\mathbf{E}^0\times\mathbf{B}_0$ frequency defined by $\omega_E=-cE^0_r/rB_0$. For purposes of analytic simplification in the stability analysis, we specialize to the case of a sharp-boundary equilibrium in which the electrons have a rectangular density profile, i.e.,

$$n_e^0(r)=\begin{cases} n_0=\text{const.}\ , & 0<r<R_p\ , \\ 0\ , & R_p<r<R_c\ , \end{cases} \tag{1}$$

where $r=R_c$ is the radial location of a grounded conducting wall. For the ion equilibrium, we make the particular choice of distribution function f_i^0 that also gives a rectangular density profile, i.e.,

$$f_i^0=(fn_0m_i/2\pi)\,\delta(H_\perp-\omega_iP_\theta-\hat{T}_i)G(v_z)\ , \tag{2}$$

where ω_i, f, and $\hat{T}_i$ are constants, $H_\perp=m_i(v_r^2+v_\theta^2)/2+e\phi^0(r)$ is the perpendicular energy, $P_\theta=m_i(rv_\theta+r^2\omega_{ci}/2)$ is the canonical angular momentum, and $G(v_z)$ is the parallel velocity *distribution* with normalization $\int_{-\infty}^{\infty} dv_zG(v_z)=1$. After some simple algebraic manipulations,[6] it is straightforward to show that the electron and ion density profiles precisely overlap with a common radius R_p provided $\hat{T}_i=m_i\Omega^2R_p^2/2$, where m_i is ion mass, $\Omega^2=\omega_E\omega_{ci}-\omega_i^2-\omega_i\omega_{ci}$, and $\omega_{ci}=eB_0/m_ic$. In the case of a sharp-boundary equilibrium, the $\mathbf{E}^0\times\mathbf{B}_0$ rotation frequency can be expressed as $\omega_E=2\pi n_0ec(1-f)/B_0$.[6]

II. ELECTROSTATIC STABILITY PROPERTIES

In the stability analysis, flute perturbations with $\partial/\partial z=0$ are considered. For perturbations with azimuthal harmonic number ℓ, a perturbed quantity $\delta\psi(\mathbf{x},t)$ can be expressed as $\delta\psi(\mathbf{x},t)=\hat{\psi}(r)\exp\{i(\ell\theta-\omega t)\}$, where ω is the complex eigenfrequency. In the electrostatic approximation with $\delta\mathbf{E}(\mathbf{x},t)=-\nabla\delta\phi(\mathbf{x},t)$, the linearized Vlasov-fluid and Poisson equations can be expressed as[6]

$$-i(\omega-\ell\omega_e)\hat{V}_{er}-(-\omega_{ce}+2\omega_e)\hat{V}_{e\theta}=(e/m_e)(\partial/\partial r)\hat{\phi}(r) ,$$

$$-i(\omega-\ell\omega_e)\hat{V}_{e\theta}+[-\omega_{ce}+(1/r)(\partial/\partial r)(r^2\omega_e)]\hat{V}_{er}=(e/m_e)(i\ell/r)\hat{\phi}(r) ,$$

$$-i(\omega-\ell\omega_e)\hat{n}_e+(1/r)(\partial/\partial r)(rn_e^0\hat{V}_{er})+(i\ell/r)n_e^0\hat{V}_{e\theta}=0 , \qquad (3)$$

$$\hat{f}_i(r,\underset{\sim}{v})=(e/m_iV_\perp)(\partial/\partial V_\perp)f_i^0[\hat{\phi}(r)+i(\omega-\ell\omega_i)\int_{-\infty}^{0}d\tau\hat{\phi}(r')\exp\{i\ell(\theta'-\theta)-i\omega\tau\}] ,$$

$$[(1/r)(\partial/\partial r)(r\partial/\partial r)-\ell^2/r^2]\hat{\phi}(r)=-4\pi e(\int d^3v\hat{f}_i-\hat{n}_e) ,$$

where the perturbation amplitudes $\hat{V}_{er}$, $\hat{V}_{e\theta}$, $\hat{n}_e$ and $\hat{f}_i$ refer to radial velocity, azimuthal velocity, electron density, and ion distribution function, respectively. Making use of Eqs. (1) and (3), and $\partial n_e^0(r)/\partial r = -n_0\delta(r-R_p)$, we obtain the eigenvalue equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1+\frac{\omega_{pe}^2}{\nu_e^2}\right)\frac{\partial}{\partial r}\hat{\phi}(r)\right]-\frac{\ell^2}{r^2}\left(1+\frac{\omega_{pe}^2}{\nu_e^2}\right)\hat{\phi}(r)$$

$$=\frac{\ell\hat{\phi}(r)}{r}\frac{\omega_{ce}-2\omega_e}{\omega-\ell\omega_e}\frac{\hat{\omega}_{pe}^2}{\nu_e^2}\delta(r-R_p) \qquad (4)$$

$$-\frac{8\pi^2e^2}{m_i}\int_0^\infty dV_\perp V_\perp\int_{-\infty}^{\infty}dv_z\frac{1}{V_\perp}\frac{\partial f_i^0}{\partial V_\perp}[\hat{\phi}(r)+(\omega-\ell\omega_i)\hat{I}] ,$$

where $\nu_e^2(r)=(\omega_{ce}-2\omega_e)^2-(\omega-\ell\omega_e)^2$=const., $\omega_{pe}^2(r)=4\pi n_0e^2/m_e$=const., for $0<r<R_p$, and $\omega_{pe}^2(r)=0$ otherwise, and orbit integral $\hat{I}$ is defined in Eq. (5).

It is evident that the perturbed electron contribution to the right-hand side of Eq. (4) [the term proportional to $\delta(r-R_p)$ in Eq. (4)] is equal to zero except at the surface of the plasma column ($r=R_p$). Moreover, it can be shown that Eq. (4) supports a class of solutions in which the perturbed ion density [the term proportional to $\int d^3v\ V_\perp^{-1}\partial f_i^0/\partial V_\perp\ldots$ in Eq. (4)] is also equal to zero except at $r=R_p$. It then follows from the linearized Poisson equation (4) that the electrostatic potential $\hat{\phi}(r)$ has the simple form $\hat{\phi}(r)=Ar^\ell$

inside the plasma column ($0 \le r < R_p$), where A is a constant. Making use of the ion trajectories in equilibrium fields, it is readily shown that the required orbit integral can be expressed as[6]

$$\hat{I} = i[\hat{\phi}(r)/(\omega_i^+ - \omega_i^-)^\ell] \int_{-\infty}^{0} d\tau \exp(-i\omega\tau)[(\omega_i - \omega_i^-)\exp(i\omega_i^+\tau) - (\omega_i - \omega_i^+)\exp(i\omega_i^-\tau)]^\ell , \tag{5}$$

where $\omega_i^{\pm} = -\omega_{ci}[1 \pm (1 + 4\omega_E/\omega_{ci})^{1/2}]/2$. An important feature of Eq. (5) is that the orbit integral $\hat{I}$ is independent of perpendicular energy $m_i V_\perp^2/2$. This is a consequence of the particularly simple form of $\hat{\phi}(r)$ within the plasma column. The right-hand side of Eq. (4) is equal to zero except at the surface of the plasma column ($r = R_p$). Moreover, the eigenfunction $\hat{\phi}(r)$ satisfies the vacuum Poisson equation, $r^{-1}(\partial/\partial r)[r\partial\hat{\phi}/\partial r] - (\ell^2/r^2)\hat{\phi}(r) = 0$, except at $r = R_p$. Therefore, the solution to Eq. (4) can be expressed as $\hat{\phi}(r) = Ar^\ell$ for $0 \le r < R_p$ and $\hat{\phi}(r) = Ar^\ell(1 - R_c^{2\ell}/r^{2\ell})/(1 - R_c^{2\ell}/R_p^{2\ell})$ for $R_p < r \le R_c$. Note that $\hat{\phi}(r)$ is continuous at $r = R_p$.

The dispersion relation that determines the complex eigenfrequency ω is obtained by multiplying Eq. (4) by r and integrating from $R_p(1-\varepsilon)$ to $R_p(1+\varepsilon)$ with $\varepsilon \to 0_+$. This gives

$$\frac{1}{1 - (R_p/R_c)^{2\ell}} = \frac{\hat{\omega}_{pe}^2}{2(\omega - \ell\omega_e)[(\omega - \ell\omega_e) - (\omega_{ce} - 2\omega_e)]} + \frac{\hat{\omega}_{pi}^2 R_p^2}{2\ell\hat{v}_i^2}\Gamma_\ell(\omega) , \tag{6}$$

where $\hat{v}_i^2 \equiv 2\hat{T}_i/m_i = \hat{r}_{Li}^2\omega_{ci}^2$, $\hat{\omega}_{pi}^2 \equiv 4\pi f n_0 e^2/m_i$, $\hat{\omega}_{pe}^2 \equiv 4\pi n_0 e^2/m_e$, $\omega_{ce} \equiv eB_0/m_e c$, and

$$\Gamma_\ell(\omega) = -1 + \left(\frac{\omega_i - \omega_i^+}{\omega_i^- - \omega_i^+}\right)^\ell \sum_{m=0}^{\ell} \frac{\ell!}{m!(\ell-m)!} \frac{\omega - \ell\omega_i}{\omega - \ell\omega_i^- - m(\omega_i^+ - \omega_i^-)} \left(\frac{\omega_i^- - \omega_i}{\omega_i - \omega_i^+}\right)^m . \tag{7}$$

A striking feature of the present analysis is the fact that the required orbit integral $\hat{I}$ [Eq. (5)] can be evaluated in closed form [Eq. (7)] for general values of the parameters $\hat{r}_{Li}/R_p$ and $(2\hat{\omega}_{pe}^2/\omega_{ce}^2)(1-f)$. Moreover, the resulting eigenvalue equation (4) for the perturbed electrostatic potential $\hat{\phi}(r)$ can be solved exactly to give a closed algebraic dispersion relation [Eq. (6)] for the complex eigenfrequency ω. As expected, in the limit where $\hat{r}_{Li}/R_p \to 0$, Eq. (6) reduces to the familiar cold-fluid dispersion relation previously discussed in the literature.[2,3]

Equation (4) has been solved numerically for the complex eigenfrequency $\omega = \omega r + i\gamma$ for a wide variety of plasma parameters.[6] For present purposes, we assume that the electron fluid is rotating in the slow equilibrium mode[1] with $\omega_e = \omega_e^- = \omega_{ce}[1-(1-4\omega_E/\omega_{ce})^{1/2}]/2$, while the mean equilibrium motion of an ion fluid element corresponds to the slow rotation velocity defined by $\omega_i = -\omega_{ci}[1-(1+4\omega_E/\omega_{ci}-4\hat{r}_{Li}^2/R_p^2)^{1/2}]/2$. The present analysis is restricted to nonneutral proton-electron plasmas ($m_i/m_e = 1836$), and the growth rate and real frequency are measured in units of the lower-hybrid frequency, $\omega_{LH} \equiv (\omega_{ce}\omega_{ci})^{1/2}$. Moreover, we assume that $R_p/R_c = 0.5$. Stability boundaries in the parameter space $(f, \omega_{pe}^2/\omega_{ce}^2)$ are illustrated in Figs. 1 and 2. In Fig. 1, the solid curves correspond to the stability boundaries ($\gamma = 0$) obtained from Eq. (6) for $\hat{r}_{Li}/R_p = 0$, and several values of azimuthal harmonic number ℓ. For a given value of ℓ, the region of $(f, \hat{\omega}_{pe}^2/\omega_{ce}^2)$ parameter space above the curve corresponds to instability ($\gamma > 0$), whereas the region of parameter space below the curve corresponds to stability ($\gamma = 0$). In the forbidden zone in Fig. 1, equilibrium is not allowed since the magnetic restoring force on an electron fluid element is weaker than the repulsive space-charge force $[2\hat{\omega}_{pe}^2(1-f) > \omega_{ce}^2]$.

In Fig. 2, the stability boundaries are illustrated for $\hat{r}_{Li}/R_p=1$. Evidently, for such large value of $\hat{r}_{Li}/R_p$, the region of $(f,\hat{\omega}^2_{pe}/\omega^2_{ce})$ parameter space corresponding to allowed equilibria becomes increasingly limited by the equilibrium constraint[6] that the pressure gradient force on an ion fluid element be weaker than the confining electric and magnetic forces (see the uppermost forbidden zone in Fig. 2).

The dependence of stability properties on fractional charge neutralization f is illustrated in Fig. 3, where the normalized growth rate γ/ω_{LH} is plotted versus f for $\hat{\omega}^2_{pe}/\omega^2_{ce}=0.002$ [Figs. 3(a) and 3(b)] and $\hat{\omega}^2_{pe}/\omega^2_{ce}=0.5$ [Fig. 3(c)], and several values of mode number ℓ. We also assume $\hat{r}_{Li}/R_p=0$ in Fig. 3(a), and $\hat{r}_{Li}/R_p=0.5$ in Figs. 3(b) and 3(c). Several features are noteworthy in Fig. 3. First, the number of unstable modes increases rapidly as f is increased. Second, for $\ell\geq2$, the instability growth rate is significantly reduced by finite ion Larmor radius effects, particularly when the equilibrium self electric field is sufficiently weak. [For example, compare Figs. 3(a) and 3(b) with f approaching unity]. Third, the instability growth rate increases substantially with increasing plasma density [Figs. 3(a) and 3(b)].

III. SUMMARY AND CONCLUSIONS

Several important conclusions follow from the present analysis. For example, the growth rate of the ion resonance instability exhibits a very sensitive dependence on $\hat{r}_{Li}/R_p$, $\hat{\omega}^2_{pe}/\omega^2_{ce}$ and f. Moreover, finite ion Larmor radius effects can have a strong stabilizing influence for mode numbers $\ell\geq2$ [see, for example, Figs. 3(a) and 3(b)], particularly when the equilibrium self-electric field is weak ($\hat{\omega}^2_{pe}/\omega^2_{ce}\ll1$ or f close to unity). For the fundamental mode ($\ell=1$), however, stability properties are identical to those calculated from a macroscopic two-fluid model, and the growth rate is unaffected by the value of $\hat{r}_{Li}/R_p$. The detailed dependence of normalized real frequency ω_r/ω_{LH} on $\hat{r}_{Li}/R_p$, $\hat{\omega}_{pe}^2/\omega_{ce}^2$ and f will be given elsewhere.[6]

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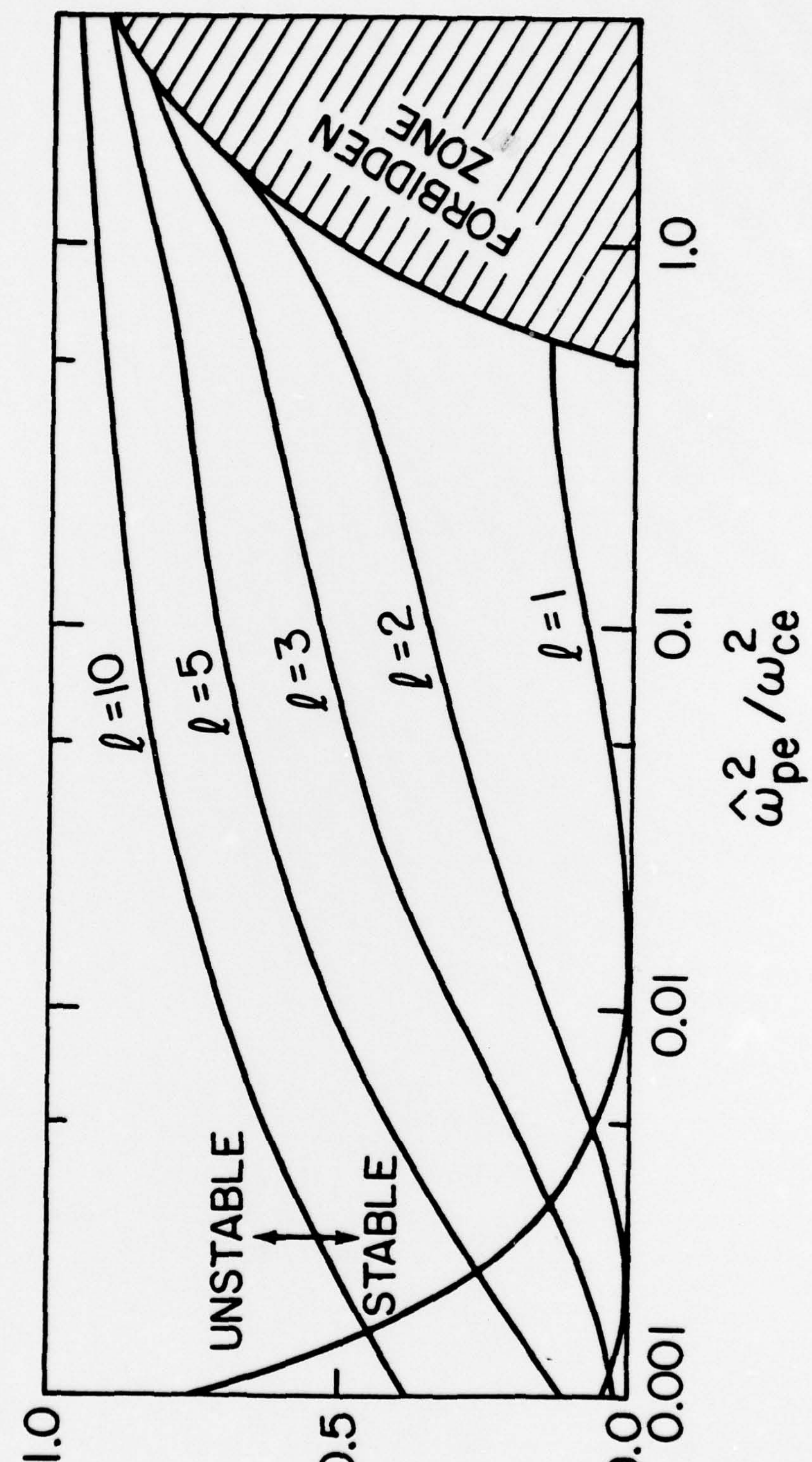
FIGURE CAPTIONS

Fig. 1 Stability boundaries [Eq. (6)] in the parameter space $(f, \hat{\omega}_{pe}^2/\omega_{ce}^2)$ for $m_i/m_e=1836$, $R_p/R_c=0.5$, $\hat{r}_{Li}/R_p=0$, and several values of ℓ.

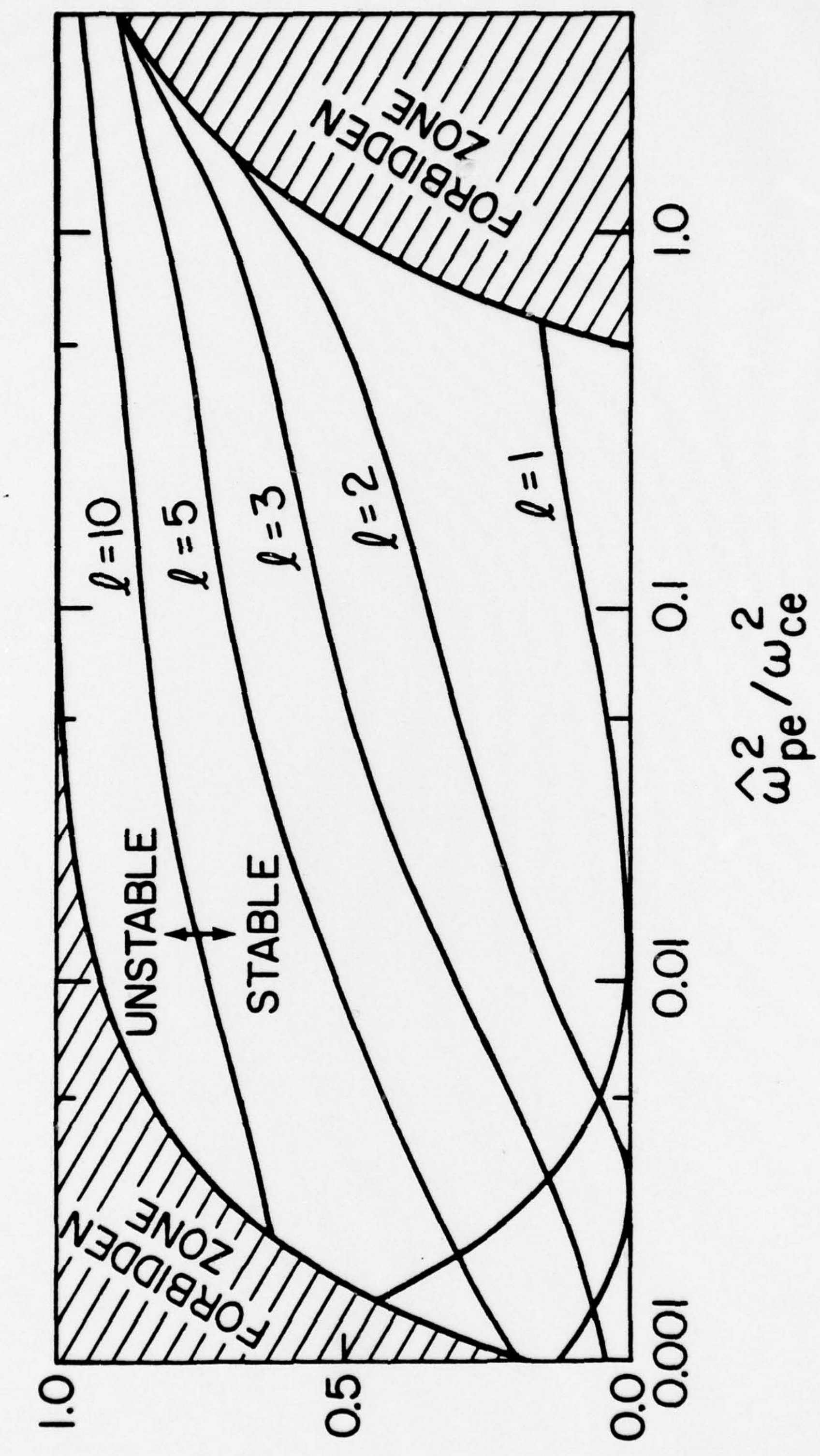
Fig. 2 Stability boundaries [Eq. (6)] in the parameter space $(f, \hat{\omega}_{pe}^2/\omega_{ce}^2)$ for $m_i/m_e=1836$, $R_p/R_c=0.5$, $\hat{r}_{Li}/R_p=1.0$, and several values of ℓ.

Fig. 3 Plots of γ/ω_{LH} versus f [Eq. (6)] for $m_i/m_e=1836$, $R_p/R_c=0.5$.



FORBIDDEN ZONE
UNSTABLE
STABLE
$\ell=10$
$\ell=5$
$\ell=3$
$\ell=2$
$\ell=1$
1.0
f 0.5
0.0
0.001
0.01
0.1
1.0
$\hat{\omega}_{pe}^2/\omega_{ce}^2$

Fig. 1



FORBIDDEN ZONE
FORBIDDEN ZONE
UNSTABLE
STABLE
$\ell=10$
$\ell=5$
$\ell=3$
$\ell=2$
$\ell=1$
f
1.0
0.5
0.0
0.001
0.01
0.1
1.0
$\hat{\omega}_{pe}^2/\omega_{ce}^2$

Fig. 2

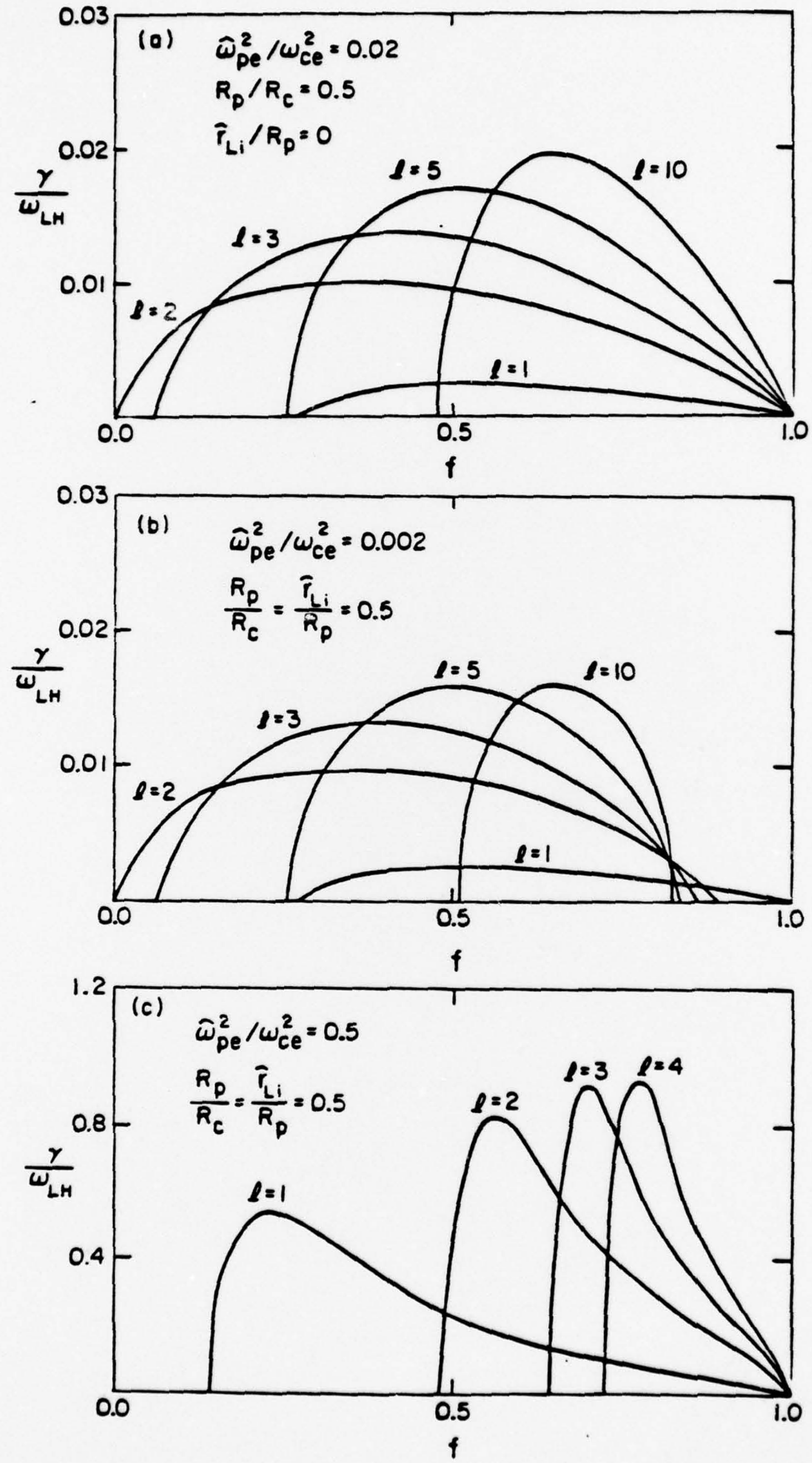

(a)
$\hat{\omega}_{pe}^2/\omega_{ce}^2 = 0.02$
$R_p/R_c = 0.5$
$\hat{r}_{Li}/R_p = 0$
$\ell = 1$
$\ell = 2$
$\ell = 3$
$\ell = 5$
$\ell = 10$
$\frac{\gamma}{\omega_{LH}}$
f
(b)
$\hat{\omega}_{pe}^2/\omega_{ce}^2 = 0.002$
$\frac{R_p}{R_c} = \frac{\hat{r}_{Li}}{R_p} = 0.5$
(c)
$\hat{\omega}_{pe}^2/\omega_{ce}^2 = 0.5$
$\frac{R_p}{R_c} = \frac{\hat{r}_{Li}}{R_p} = 0.5$
$\ell = 4$

Fig. 3